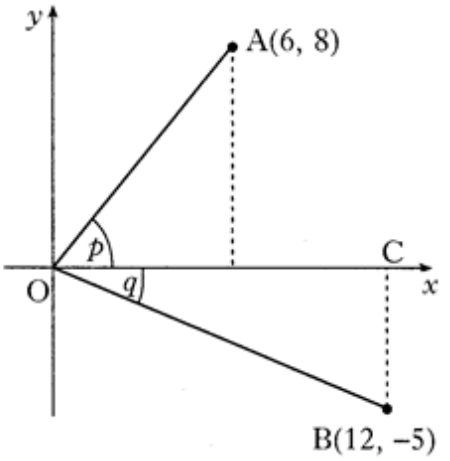
Trigonometric Formulae

2000 P1

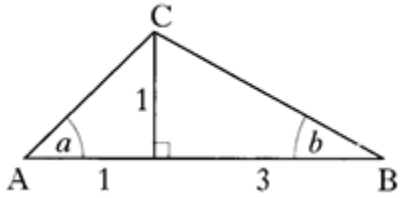
A1. On the coordinate diagram shown, A is the point (6, 8) and B is the point (12, −5). Angle AOC = p and angle COB = q.

Find the exact value of $\sin(p + q)$. **4**



2002 P1

5. In triangle ABC, show that the exact value of $\sin(a+b)$ is $\frac{2}{\sqrt{5}}$. **4**



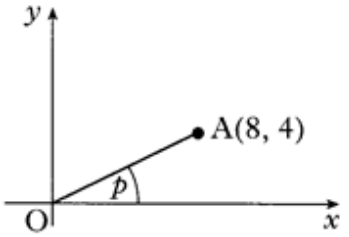
2003 P1

10. A is the point (8, 4). The line OA is inclined at an angle p radians to the x-axis.

(a) Find the exact values of:

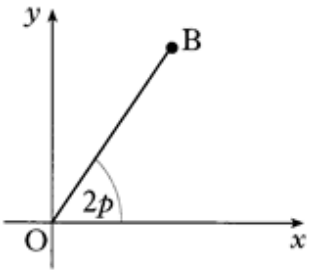
(i) $\sin(2p)$;

(ii) $\cos(2p)$. **5**



The line OB is inclined at an angle $2p$ radians to the x-axis.

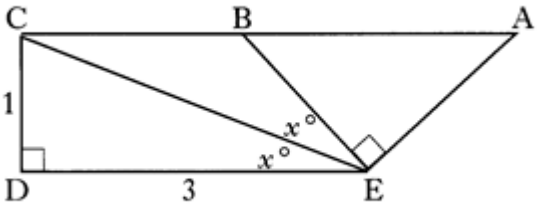
(b) Write down the exact value of the gradient of OB. **1**



2004 P1

10. In the diagram
angle DEC = angle CEB = $x°$ and
angle CDE = angle BEA = 90°.
CD = 1 unit; DE = 3 units.

By writing angle DEA in terms of $x°$, find the exact value of $\cos(D\hat{E}A)$. **7**

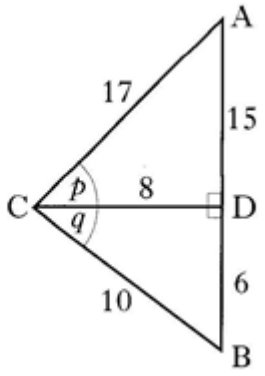


2005 P1

9. If $\cos 2x = \frac{7}{25}$ and $0 < x < \frac{\pi}{2}$, find the exact values of $\cos x$ and $\sin x$. **4**

2005 P2

2. Triangles ACD and BCD are right-angled at D with angles p and q and lengths as shown in the diagram.



(a) Show that the exact value of $\sin(p+q)$ is $\frac{84}{85}$. **4**

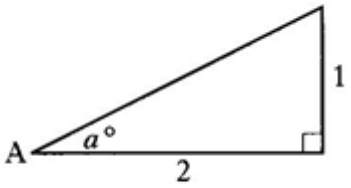
(b) Calculate the exact values of:

(i) $\cos(p+q)$;

(ii) $\tan(p+q)$. **3**

2006 P2

8. The diagram shows a right-angled triangle with height 1 unit, base 2 units and an angle of $a°$ at A.



(a) Find the exact values of:

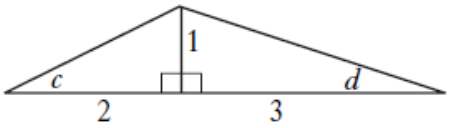
(i) $\sin a°$;

(ii) $\sin 2a°$. **4**

(b) By expressing $\sin 3a°$ as $\sin(2a+a)°$, find the exact value of $\sin 3a°$. **4**

2007 P2

2. The diagram shows two right-angled triangles with angles c and d marked as shown.



(a) Find the exact value of $\sin(c+d)$. **4**

(b) (i) Find the exact value of $\sin 2c$.

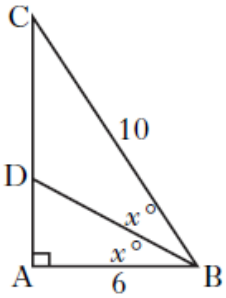
(ii) Show that $\cos 2d$ has the same exact value. **4**

2008 SP1 P2

7. Triangle ABC is right-angled at A and BD is the bisector of angle ABC.

AB = 6 units and CB = 10 units.

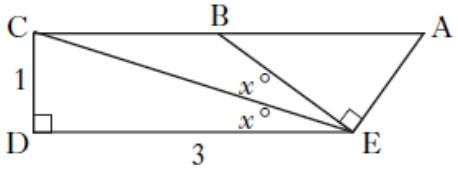
Determine the exact value of BD, expressing your answer in its simplest form. **5**



2008 SP2 P1

24. In the diagram,

angle DEC = angle CEB = $x°$ and
angle CDE = angle BEA = 90°.
CD = 1 unit; DE = 3 units.



By writing angle DEA in terms of $x°$, find the exact value of $\cos(\hat{DEA})$. **7**

2009 P1

24. (a) Using the fact that $\frac{7\pi}{12}=\frac{\pi}{3}+\frac{\pi}{4}$, find the exact value of $\sin\left(\frac{7\pi}{12}\right)$. **3**

(b) Show that $\sin(A+B)+\sin(A-B)=2\sin A\cos B$. **2**

(c) (i) Express $\frac{\pi}{12}$ in terms of $\frac{\pi}{3}$ and $\frac{\pi}{4}$.

(ii) Hence or otherwise find the exact value of $\sin\left(\frac{7\pi}{12}\right)+\sin\left(\frac{\pi}{12}\right)$. **4**

2010 P1

23. (*a*) Diagram 1 shows a right angled triangle, where the line OA has equation $3x - 2y = 0$.

(i) Show that $\tan a = \frac{3}{2}$.

(ii) Find the value of $\sin a$. **4**

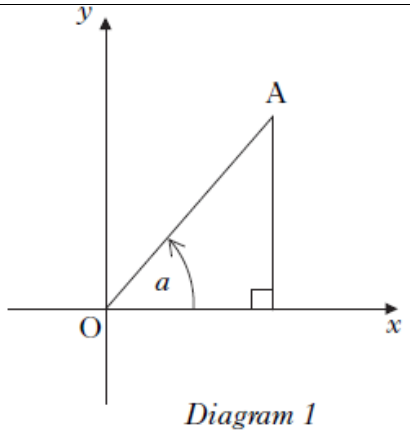


Diagram 1

(*b*) A second right angled triangle is added as shown in Diagram 2.

The line OB has equation $3x - 4y = 0$.

Find the values of $\sin b$ and $\cos b$. **4**

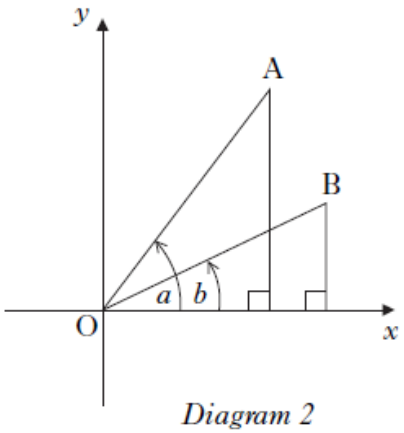


Diagram 2

(*c*) (i) Find the value of $\sin(a - b)$.

(ii) State the value of $\sin(b - a)$. **4**

2015 SP P1

6. (a) Find an equivalent expression for $\sin(x + 60)^\circ$. **1**

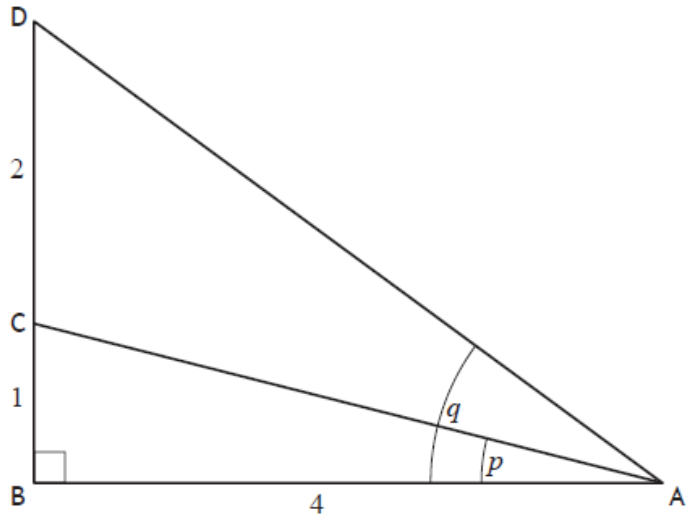
(b) Hence, or otherwise, determine the exact value of $\sin 105^\circ$. **3**

2015 P1

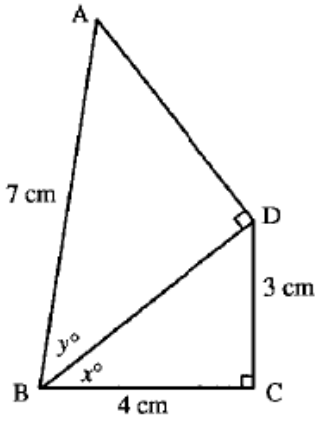
10. Given that $\tan 2x = \frac{3}{4}$, $0 < x < \frac{\pi}{4}$, find the exact value of

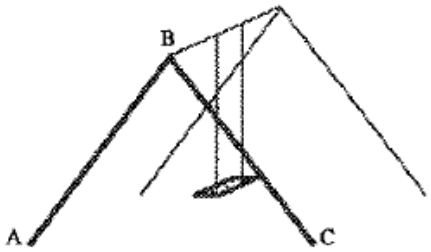
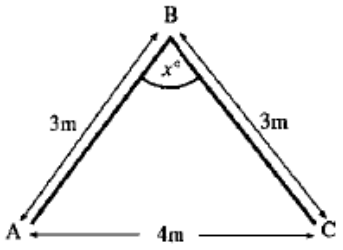
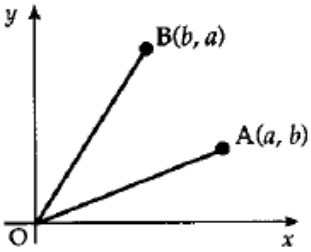
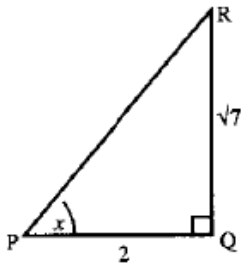
(a) $\cos 2x$ **1**

(b) $\cos x$. **2**

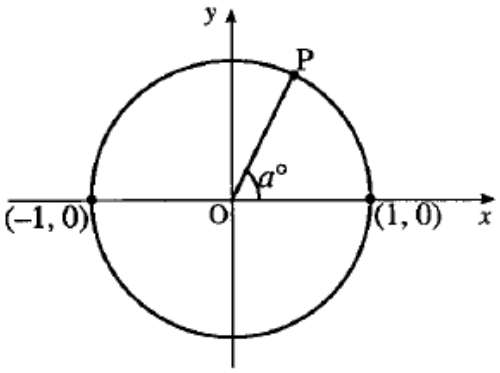
2016 P1	13. Triangle ABD is right-angled at B with angles BAC = p and BAD = q and lengths as shown in the diagram below.  Show that the exact value of $\cos(q-p)$ is $\frac{19\sqrt{17}}{85}$.	5

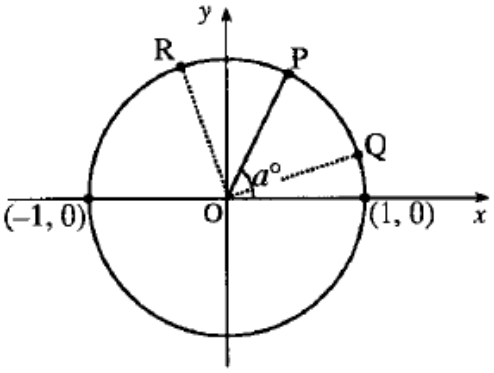
Pre 2000 - Trigonometric Formulae

1	A and B are acute angles such that $\tan A = \frac{3}{4}$ and $\tan B = \frac{5}{12}$. Find the exact value of (*a*) $\sin 2A$ (*b*) $\cos 2A$ (*c*) $\sin(2A + B)$.	2 1 2
2	If $x°$ is an acute angle such that $\tan x° = \frac{4}{3}$, show that the exact value of $\sin(x+30)°$ is $\frac{4\sqrt{3}+3}{10}$.	3
3	The diagram shows two right-angled triangles ABD and BCD with AB = 7cm, BC = 4cm and CD = 3cm. Angle DBC = $x°$ and angle ABD = $y°$. Show that the exact value of $\cos(x+y)°$ is $\frac{20-6\sqrt{6}}{35}$.  	3

4	The framework of a child's swing has dimensions as shown in the diagram on the right. Find the exact value of $\sin x°$.   	5
5	Given that $\tan\alpha = \frac{\sqrt{11}}{3}$, $0 < \alpha < \frac{1}{2}\pi$, find the exact value of $\sin 2\alpha$.	3
6	Find algebraically the **exact** value of $\sin\theta^\circ + \sin(\theta+120)^\circ + \cos(\theta+150)^\circ$.	3
7	If $\cos\theta = \frac{4}{5}$, $0 \le \theta < \frac{\pi}{2}$, find the **exact** value of (*a*) $\sin 2\theta$ (*b*) $\sin 4\theta$.	 2 3
8	For acute angles P and Q, $\sin P = \frac{12}{13}$ and $\sin Q = \frac{3}{5}$. Show that the **exact** value of $\sin(P+Q)$ is $\frac{63}{65}$.	3
9	Given that $\sin A = \frac{3}{4}$, where $0 < A < \frac{\pi}{2}$, find the **exact** value of sin 2A.	3
10	Given that $\cos D = \frac{2}{\sqrt{5}}$ and $0 < D < \frac{\pi}{2}$, find the exact values of sin D and cos 2D.	3
11	In the diagram, A and B have coordinates as shown. Express $\sin A\hat{O}B$ in terms of *a* and *b*.  	4
12	Using triangle PQR, as shown, find the exact value of cos 2*x*.  	3

13 **The diagram shows a circle of radius 1 unit and centre the origin. The radius OP makes an angle $a°$ with the positive direction of the x-axis.**





(*a*) Show that P is the point $(\cos a°, \sin a°)$. **1**

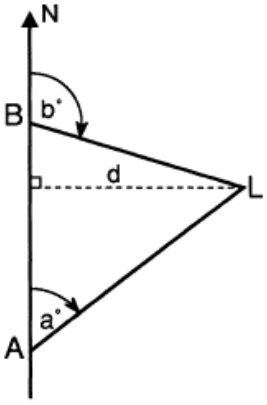
(*b*) If $\hat{POQ} = 45°$, deduce the coordinates of Q in terms of a. **1**

(*c*) If $\hat{POR} = 45°$, deduce the coordinates of R in terms of a. **1**

(*d*) Hence find an expression for the gradient of QR in its simplest form. **4**

(*e*) Show that the tangent to the circle at P is parallel to QR. **2**

14 A ship is sailing due north at a constant speed. When at position A, lighthouse L is observed on a bearing of $a°$. One hour later, when the ship is at position B, the lighthouse is on a bearing of $b°$. The shortest distance between the ship and the lighthouse during this hour was d miles.



(*a*) Prove that $AB = \dfrac{d}{\tan a°} - \dfrac{d}{\tan b°}$. **(2)**

(*b*) Hence prove that $AB = \dfrac{d\sin(b-a)°}{\sin a°\sin b°}$. **(3)**

(*c*) Calculate the shortest distance from the ship to the lighthouse when the bearings $a°$ and $b°$ are 060° and 135° respectively and the constant speed of the ship is 14 miles per hour. **(3)**